A Mathematical Approach to Investigate the Effect of Chip Slenderness Ratio in Primary Deformation Zone in Turning

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Abstract—In extended cutting speed law, Chip Slenderness Ratio (Csr) is a vital parameter and ratio of the depth of cut to feed rate, influencing machining operations. In this study, the effect of Chip Slenderness Ratio (χ) on the shear angle (φ), cutting-edge approach angle (α), and rake angle (γ) was investigated theoretically, by using Matlab and Minitab 17 software. In conclusion, it established that φ has an inversely proportional effect on Csr, while χ has a directly proportional effect, but γ has an insignificant effect on primary deformation zone because of affecting moving of removed chip on rake face.

Index Terms— chip slenderness ratio, shear angle, cutting edge approach angle, primary deformation zone, orthogonal cutting.

1 INTRODUCTION

MACHINING method widely used as a final stage in manufacturing operations. Machinability depends on many different kinds of parameters, such as cutting speed, feed rate and depth of cut, geometrical and structural properties of the tool, and the sample material properties. In addition to these parameters, in the extended cutting speed law, the effect of the Chip Slenderness Ratio (Csr) is taken into consideration as an influential parameter in machining operations. G is the ratio of the depth of cut to the feed rate. Furthermore, this ratio (Csr) is identified as an equation, depending on chip cross-sectional area (A) in extended cutting speed law. In other words, the higher the Slenderness Ratio (Csr), the higher ratio of cutting depth to the feed rate for the same volumetric chip removal rate. 5 is the most frequently used value in turning operations as a Chip Slenderness Ratio (Csr), in which the selected feed rate is not recommended to be higher than the depth of cut. Therefore, 5 can take into consideration, as a standard value for Csr, due to using widely in practice. Moreover, cutting speed and thereby cutting force consisting of values can be attributed to Csr. Additionally; the effect of cross-sectional area (A) on the cutting speed is wider than the slenderness ratio (Csr). When chip slenderness ratio (Csr) values are between 5:1 and 20:1, there is an increase in cutting speed by 21.5%. The cutting speed diversifies about ±9 when the cutting edge angle varied from 0° to 60°. However, the larger cutting edge angle values mostly occur in face milling operations [1].

Chip removal rate is a vital conclusion in machining processes according to machinability standard. It, derived from multiplying feed rate, depth of cut, and cutting speed, is a vital parameter in manufacturing areas of mass production [2]. High cutting speed, feed rate, and depth of cut make the material removal rate increased linearly. However, the effect of cutting speed is greater than the others. An increase in cutting speed at any selected feed rate provides the material removal rate to increase significantly, but it increases slightly with increasing depth of cut, even at higher selected feed rates. Moreover, the most influential parameter on material removal rate is spindle speed, followed by feed rate and depth of cut, respectively [3]. While cutting speed provides higher material removal rate, depth of cut causes tool chatter, reducing material removal rate. Furthermore, higher cutting depth creates higher thrust force on the cutting tool, launching the tool radially outwards and as a result, causing vibrations, and thus leading to a reduction in material removal rate.

According to these consequences, the main parameter, contributing to control material removal rate, is feed rate [4]. Depending on Chip Slenderness Ratio (Csr), the depth of cut is a vital parameter for the material removal rate in turning operations [5]. However, the effect of cutting speed and depth of cut on feed and radial forces is higher than the feed rate in turning [6]. The residual stresses in turning become larger with increasing feed rate, but the depth of cut does not have any significant effect [7]. Chip morphology influenced by the selected parameters; especially, feed rate and depth of cut cause saw-teeth chip formation as well as cutting speed and cutting-edge radius [9]. Moreover, at higher selected feed rates, removed chip occurs in massive segments. Cutting speed is a vital parameter to cause tool wear and make the machined material harden [9]. Furthermore, greater feed rate, but the lower depth of cut provides the lowest the process temperature. However, higher selected depths of cut cause lower processes temperature. Furthermore, tool temperature decreases at the selected greater feed rate, but the lower depth of cut and greater depth of cut, but lower feed rate during the operation [10]. Therefore, by altering the cutting depth, tool life increases, so the consumed energy load of the machine and cutting forces decrease [11]. Additionally, Cutting force increases with increasing feed rate, but it decreases with increasing cutting speed [12]. Moreover, selecting cutting depth is a vital phenomenon in turning because it affects the cost of the process [13]. Although the depth of cut does not influence tool wear significantly, in continuous turning operations, different depths of cut shorten the tool life [14]. At a lower selected depth of cut values, higher tool wear takes place [15].

Many different parameters, making difficult to control the dimensional precision in machining operations, have an influence on the residual stress and distortion. A vital parameter among them is the depth of cut, affecting surface roughness, but there is not a linear proportion between the depth of cut and residual stress [16]. Furthermore, Surface roughness is affected by spindle speed and feed rate directly. With increasing feed rate, surface roughness increases linearly, but it decreases with increasing spindle speed [17].

The main purpose of this paper is to investigate the effect of Chip Slenderness Ratio (Csr) on shear, rake, and cutting-edge approach angle, by using the mathematical and geometrical approach in primary deformation zone.
2. GEOMETRY OF ORTHOGONAL CUTTING MECHANISM IN PRIMARY DEFORMATION ZONE

In this study, the orthogonal cutting mechanism is analysed mathematically and geometrically in the primary deformation zone. During the modelling process, the effect of Chip Slenderness Ratio (Csr) on shear, rake, and cutting-edge approach angle was investigated by using the mathematical and geometrical equation, derived from the orthogonal cutting mechanism according to chip removal rate (CRr). A derived equation analysed by using Matlab and Minitab software. With the help of these software programs, the relationship between Chip Slenderness Ratio (Csr) - shear, rake, and cutting-edge approach angles was investigated, by gaining graphs.

The material removal rate is a vital conclusion in machining operations, affecting the selected parameters, especially cutting speed, depth of cut and feed rate. Machinability can be evaluated depending on these parameters. In primary deformation zone, when the chip removed from the machined material, the process results are affected by different kinds of parameters according to the orthogonal cutting mechanism. One of the most important of these parameters is Chip Slenderness Ratio (Csr) in extended cutting speed law. Therefore, the effect of this parameter (Csr) on shear, rake, and cutting-edge approach angle was investigated mathematically and geometrically depending on the orthogonal cutting mechanism.

2.1 Mechanics of Chip Formation in Turning

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Chip removal rate (Csr) obtained by multiplying cutting speed, depth of cut, and feed rate. Material removal rate can calculate in the volumetric unit in one cycle of material or in an identified time, such as one second or one minute in turning operations as demonstrated in Fig. 1.

Material removal rate can be calculated geometrically, depending on shear, rake, and cutting-edge approach angle, associated with depth of cut, cutting speed, namely spindle speed, and feed rate, as in Equation 1 in one cycle of sample material, as seen in Fig. 1.

\[ C_{Rr} = \alpha . f . V \]  

(1)

Furthermore, cutting speed (V) is calculable depending on spindle speed and diameter of the cylindrical machined sample during one minute time as a period, according to Equation 2, also demonstrated in Fig. 1 as (round per minute) rpm.

\[ V_m = \frac{\pi . d . n}{1000} (\text{m/min}) \]  

(2)

Additionally, cutting speed (V) equation can be written depending on the diameter of sample and spindle speed in one second as revolutions in a frequency, as shown in Fig. 1; and as (round per second) rps, can be written as in Equation 3.

\[ V = \frac{3 . \pi . d . n . f . a}{50} (\text{m/s}) \]  

(3)

Therefore, chip removal rate (CRr) is derivable depending on cutting speeds of in one minute and in one second, as demonstrated in Equation 4.

\[ C_{Rr} = \frac{\pi . d . n . f . a}{1000} (\text{mm}^3/\text{min}) \Rightarrow C_{Rr} = \frac{3 . \pi . d . n . f . a}{50} (\text{mm}^3/\text{s}) \]  

(4)

2.2 The Effect of Chip Slenderness Ratio (Csr) in Primary Deformation Zone

Machinability mainly depends on the shear and tool cutting edge approach angles, associated with other parameters, selected by operators in primary deformation zone. Especially, one of most important of these parameters is Chip Slenderness Ratio (Csr), the ratio of the depth of cut to the feed rate. By means of the orthogonal cutting mechanism, the effect of Csr on shear, rake and cutting edge approach angles was investigated theoretically by using cutting geometry as seen in Fig. 2.

The chip removed from the sample (t_c) can be identified as demonstrated in Equation 5, depending on cutting-edge approach angle and feed rate.

\[ t_c = f . \cos(\gamma) (\text{mm}) \]  

(5)

Additionally, removed chip (t_c) is derivable as in Equation 6, depending on uncut chip thickness (t), shear angle (\( \varphi \)), and rake angle (a).

\[ C_{Rr} = \frac{\pi . d . n . f . a}{1000} (\text{mm}^3/\text{min}) \Rightarrow C_{Rr} = \frac{3 . \pi . d . n . f . a}{50} (\text{mm}^3/\text{s}) \]  

(4)
Vertical plane (ε), identified with lines in green colour, is normal to the main plane of the cutting tool. When the removed chip leaves in the primary deformation zone, then moves away on the rake face of the tool, under an inclination, denoting (α), with the vertical plane (ε). The chip is removed from the sample with shear speed (VS) and cutting speed (V), then it moves away from the zone on the rake face with (VC) speed. Additionally, removed chip (tc) is derivable as in Equation 6, depending on uncut chip thickness (t), shear angle (φ), and rake angle (α).

\[
|\text{DC}| = \frac{a}{\cos(\chi)} \quad \text{(mm)}
\]

(11)

The volume of the chip removal rate, circumscribed with |ABCDEFGH| points, is calculable with the help of the Jacobian method, as specified in Equations 12, 13, and 14, as shown in Fig. 2.

\[
J_v = \begin{vmatrix}
\frac{d(AD)}{d\phi} & \frac{d(AD)}{dt} & -t\cos(\phi) & 1 \\
\frac{d(DC)}{d\chi} & \frac{d(DC)}{d\chi} & a\sin(\chi) & 1 \\
\end{vmatrix}
\]

\[
\cos(\chi) \quad \sin(\phi) \quad \cos(\chi) 
\]

\[
C_{Rr} = \iiint J_v d\phi d\chi dV
\]

(13)

3 RESULTS AND DISCUSSION

3.1 The Relationship between C_{sr} and φ, χ, α Angles

Chip Slenderness Ratio (Csr) is computable depending on shear angle (φ), cutting-edge approach angle (χ), and rake angle (α).
The relationship among the Chip Slenderness Ratio (Csr), the shear (φ), cutting-edge approach (χ), and rake (α) angles, as shown in Fig. 3 a, b, and c, respectively. By using Matlab software, the graphs of Csr and φ, χ, α were achieved as seen in Fig. 3, 4, and 5, respectively according to Equation 17. For the algorithm, by applying Matlab software, shear angle (φ), cutting-edge approach angle (χ), and rake angle (α) were selected between $0^\circ$ – $45^\circ$, $0^\circ$ – $45^\circ$, and $0^\circ$ – $20^\circ$, respectively. Equalizing of Csr to 5 is standard in practice machining operations, especially in turning [1]. The graphs of the relationship between shear angle (φ) and Csr demonstrated as in Fig. 3. According to this graph, shear angle (φ) specified in radian unit. If we multiply values on the shear angle (φ) axis with $180/\pi$, the angle values are obtainable in degree unit. According to the graph, as demonstrated in Fig. 3, there is an inversely proportional relationship between the shear angle (φ) and Chip Slenderness Ratio (Csr). According to the graph, the optimum Chip Slenderness Ratio (Csr=5) shear angle is equal to about $15^\circ$. When the shear angle (φ) decreases Csr value increases linearly. According to the standard machinability criterion, shear angle values are between $150$ and $45^\circ$, $(15^\circ \leq \phi \leq 45^\circ)$, Chip Slenderness Ratio (Csr) is smaller than 5 (Csr$<5$). With changing shear angle values between $0^\circ$ – $45^\circ$, associated with the rake (α) and cutting-edge approach (χ) angles, Csr takes values smaller than 40 and bigger than 0, $(0 \leq \text{Csr} \leq 40)$. Although there is an inverse proportion between Csr and φ, there is a directly proportional relationship between cutting edge angle (χ) and Chip Slenderness Ratio (Csr).

However, the inclination of the curve of this relationship changes associated with the shear angle (φ) and rake angle (α) as seen in Fig. 4. For the standard Csr values, smaller than 5, cutting-edge approach angle (χ) takes values approximately bigger than $6^\circ$. At smaller cutting-edge approach angle (χ) Csr values are bigger than 5, as demonstrated in Fig. 4. Contrary to shear (φ) and cutting-edge approach (χ) angles, rake angle (α) has an insconsiderable impact on Chip Slenderness ratio (Csr). With changing rake angle (α) between $0^\circ$ – $20^\circ$, Csr changes about 1 unit as seen in Fig. 5. This result shows that rake angle (α) does not have an insignificant effect on Csr in primary deformation zone. Because rake angle (α) affects the moving of removed chip on the rake face of the cutting tool, thus it does not have any significant effect in primary deformation zone.

3.2 The Relationship Between Csr and φ, χ, α Angles

Furthermore, the relationship between the Chip Slenderness Ratio (Csr) and shear (φ), cutting-edge approach (χ), and rake (α) angles investigated using with a Minitab 17 software pro-
gram. This relationship demonstrated in Fig. 6. This relationship shows an alteration compatible with curves, achieved by using the Matlab software. In Fig. 6, the relationship between Csr and χ is demonstrated with three different kinds of curves, also the relationship of α with two kinds of curves, while φ showed with only one curve. As interpreted under 3.1 title, with increasing shear angle (φ), Chip Slenderness Ratio (Csr) decreases, while it increases with increasing cutting tool approach angle (χ), but there is not any insignificant rake angle (α) on Chip Slenderness Ratio (Csr).

![Fig. 6. The relationship of Chip Slenderness Ratio (Csr), with shear angle (φ), cutting edge approach angle (χ), and rake angle (α).](image)

## 4 Conclusion

Although Chip Slenderness Ratio (Csr) has a vital influence on machining operations, in extended cutting speed, it has scarcely studied by researchers in literature. Especially, it has a vital impact on the shear angle (φ) and cutting-edge approach angle (χ), in primary deformation zone, though it has an inconsiderable effect on rake angle (α). There is an inversely proportional relationship between the shear angle (φ) and Chip Slenderness Ratio (Csr), while there is a direct proportion relationship with cutting edge approach angle (χ).

However, rake angle (α) has an insignificant impact on Chip Slenderness Ratio (χ) because rake angle (α) affects only the moving of removed chip on the rake face of the cutting tool in secondary deformation zone. The optimum shear angle (φ) is about 15o according to the optimum Chip Slenderness Ratio (Csr), which is equal to 5. Moreover, increases in shear angle (φ) cause decreasing Csr smaller than 5, and also invertibility is valid. However, with increasing cutting edge approach angle, Csr increases linearly, but the inclination of the curve decreases with decreasing χ associated with other selected parameters.

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### Acknowledgment
A, B, C. This work was not supported in part by any one and/or any sector.

### References


